# Projectile Motion

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## Projectile Motion

- **Projectiles**
  - > **objects given an initial velocity that then move under the force of gravity**
- **Trajectory**
  - > **the path followed by a projectile**
  - > **The path is a curve called a parabola**

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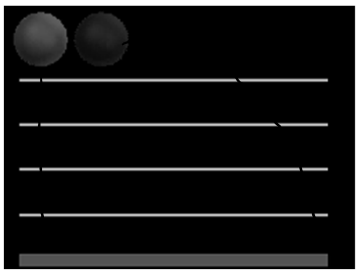
# Independence of Dimensions



FALLING FROM SAME HEIGHT

GRAVITY IS ONLY THING PULLING BOTH DOWN

**Why do both balls hit at the same time?**

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# Independence of Dimensions

- **Since the horizontal and vertical motion of an object are independent of each other, the motion equations can be used to determine the exact position of a projectile.**
- **However, we must first distinguish between the x and y components of any vectors.**

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# Independence of Dimensions

- With no acceleration in the horizontal direction, we can find the horizontal position by using the equation:
  - $X = X_0 + V_{0x}t + \frac{1}{2}At^2 \Rightarrow X = V_{0x}t$
- The velocity in the horizontal direction will not change, therefore:
  - $V_x = V_{0x} + A_x t \Rightarrow V = V_0$

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# Independence of Dimensions

- Since there is acceleration (gravity) in the vertical direction the position can be found using the equation:
  - $Y = Y_0 + V_{0y}t + \frac{1}{2}A_y t^2$
- The acceleration causes a change in velocity in the vertical direction. We can find the final velocity using the equations:
  - $V_y = V_{0y} + A_y t$
  - $V_y^2 = V_{0y}^2 + 2A_y(Y - Y_0)$

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# Projectile Problem

- **A stone is thrown horizontally at a speed of 15 m/s from the top of a cliff 78.4 m high.**
  - > **How long is the stone in the air?** $t$
  - > **How far from the cliff does the stone land?** $X$
  - > **What is the horizontal and vertical components of the velocity just before the stone hits the ground?** $V_x, V_y$

SAME FOR ALL HORIZONTAL PROBLEMS

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 78.4 |
| $v_0$ | 15 | 0 |
| v | 15 | |
| a | 0 | -9.8 |
| t | | |

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# Projectile Problem

- **Find the time** USE THE COLUMN WITH BOTH DISTANCES

$Y = Y_o + V_{o_y}t + \frac{1}{2}A_y t^2$

$0 = 78.4 + (0)(t) + \frac{1}{2}(-9.8)t^2$

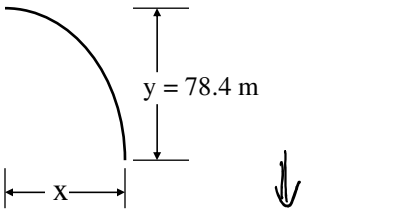
$0 = 78.4 + (-4.9)t^2$

$-78.4 = (-4.9)t^2$

$\frac{-78.4}{-4.9} = t^2$

$16 = t^2$

$4_s = t$




| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 78.4 |
| $v_0$ | 15 | 0 |
| v | 15 | |
| a | 0 | -9.8 |
| t | | |

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# Projectile Problem

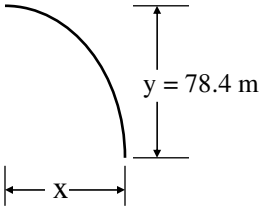
- **Find the horizontal distance**

↳ Use x column

$x = v_0 t$

$x = 15(4)$

$x = 60\ m$




| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 78.4 |
| $v_0$ | 15 | 0 |
| v | 15 | |
| a | 0 | -9.8 |
| t | 4 ⇔ SAME | 4 |

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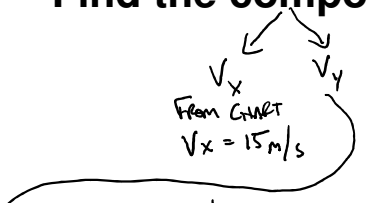
# Projectile Problem

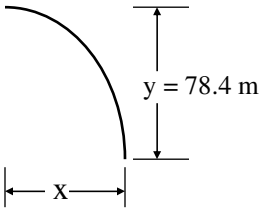
- **Find the components**




$V_y = V_{oy} + At$

$V_y = 0 + -9.8(4)$

$V_y = -39.2\ m/s$




| | X | Y |
|---|---|---|
| x or y | 0 | 0 |
| $x_o$ or $y_o$ | 60 | 78.4 |
| $v_0$ | 15 | 0 |
| v | 15 | |
| a | 0 | -9.8 |
| t | 4 | 4 |

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# Horizontal Projectile Problem #2

- **Jordan throws a dart horizontally at a dart board 2.2 meters away. If the initial velocity of the dart is 3.6 m/s, how far does the dart drop before hitting the board?**

| | X | Y |
|---|---|---|
| x or y | 2.2 | 0 |
| $x_o$ or $y_o$ | 0 | |
| $v_0$ | 3.6 | 0 |
| v | 3.6 | |
| a | 0 | -9.8 |
| t | | |

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# Horizontal Projectile Problem #2

FIND TIME → USE COLUMN WITH BOTH DISTANCES

$X = V_{0x}t$

$2.2 = 3.6t$

$\frac{2.2}{3.6} = t$

$t = .61s$

FIND Y

$y = y_0 + v_{0y}t + \frac{1}{2}A_y t^2$

$y = 0 + \cancel{0(.61)}^{0} + \frac{1}{2}(-9.8)(.61)^2$

$y = 1.8_m$

| | X | Y |
|---|---|---|
| x or y | 2.2 | 0 |
| $x_o$ or $y_o$ | 0 | |
| $v_0$ | 3.6 | 0 |
| v | 3.6 | |
| a | 0 | -9.8 |
| t | .61 | .61 |

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# Horizontal Projectile Problem #3

- **You are at the top of a 3.5 m high stair case. A friend at the bottom of the stairs forgot her pencil and asks you for one. You notice that she is 8.5 m horizontally away from you. What horizontal velocity should you throw the pencil at to ensure that she gets the pencil?**

| | X | Y |
|---|---|---|
| x or y | 8.5 | 0 |
| $x_o$ or $y_o$ | 0 | 3.5 |
| $v_0$ | | 0 |
| v | | |
| a | 0 | -9.8 |
| t | | |

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# Horizontal Projectile Problem #3

FIND TIME => USE COLUMN WITH MOST NON-ZERO INFORMATION

$Y = Y_0 + V_{0y}t + \frac{1}{2}A_y t^2$

$0 = 3.5 + (0)t^0 + \frac{1}{2}(-9.8)t^2$

$0 = 3.5 + (-4.9)t^2$

$-3.5 = -4.9t^2$

$\frac{-3.5}{-4.9} = t^2$

$.714 = t^2$

$t = .84s$

FIND VELOCITY

$X = V_x t$

$8.5 = V_x(.84)$

$\frac{8.5}{.84} = V_x$

$V_x = 10.1 m/s$

| | X | Y |
|---|---|---|
| x or y | 8.5 | 0 |
| $x_o$ or $y_o$ | 0 | 3.5 |
| $v_0$ | | 0 |
| v | | |
| a | 0 | -9.8 |
| t | .84s | .84s |

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# Projectiles Launched at an Angle

- When projectiles are launched at an angle, they are given an initial horizontal and vertical velocity.
- The horizontal distance the projectile travels is called the range.


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# Angled Launch Problem #1

- A ball is thrown with a initial velocity of 5.5 m/s at an angle of $54^0$. Find:
  - > the time in the air.
  - > how high the ball went.
  - > what the range was.

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | | |
| v | | |
| a | 0 | -9.9 |
| t | | |
| $t_{max}$ | — | |
| $\Delta y_{max}$ | — | |

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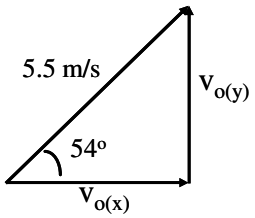
# Angled Launch Problem #1

- **Find components**

SOH CAH TOA




$V_{0x}$ => ADJACENT => CAH

$\cos 54° = \frac{V_{0x}}{5.5}$

$V_{0x} = 5.5 \cos 54° = 3.23\ m/s$

$V_{0y}$ = OPPOSITE => SOH

$\sin 54° = \frac{V_{0y}}{5.5}$

$V_{0y} = 5.5 \sin 54° = 4.45$

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | 3.23 | 4.45 |
| v | 3.23 | -4.45 |
| a | 0 | -9.8 |
| t | | |
| $t_{max}$ | — | |
| $\Delta y_{max}$ | — | |

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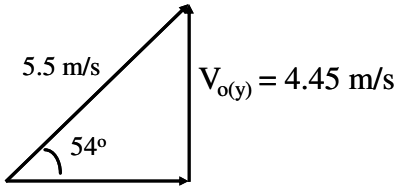
# Angled Launch Problem #1

- **Find time**




$V_{o(x)} = 3.23$ m/s

$V_y = V_{0y} + A_y t$

$-4.45 = 4.45 + -9.8t$

$-8.90 = -9.8t$

$\frac{-8.9}{-9.8} = t$

$t = .91s$ => $t_{max}$ IS HALF OF THE TOTAL TIME

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | 3.23 | 4.45 |
| v | 3.23 | -4.45 |
| a | 0 | -9.8 |
| t | .91 | .91 |
| $t_{max}$ | — | .455 |
| $\Delta y_{max}$ | — | |

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# Angled Launch Problem #1

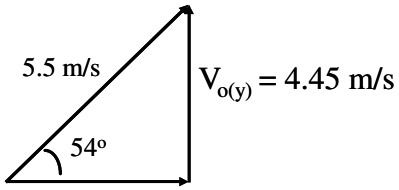
- **Find Max height**

HAPPENS AT HALF THE TOTAL TIME

$y_{max} = y_0 + v_0 t_{max} + \frac{1}{2} a_y t_{max}^2$

$y_{max} = 0 + (4.45)(.455) + \frac{1}{2}(-9.8)(.455)^2$

$y_{max} = 1.01\ m$




$V_{o(x)} = 3.23$ m/s

| | X | Y |
|---|---|---|
| x or y | 0 | 0 |
| $x_o$ or $y_o$ | | 0 |
| $v_0$ | 3.23 | 4.45 |
| v | 3.23 | -4.45 |
| a | 0 | -9.8 |
| t | .91 | .91 |
| $t_{max}$ | — | .455 |
| $\Delta y_{max}$ | — | 1.01 |

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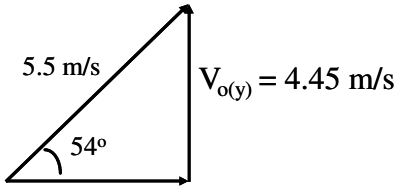
# Angled Launch Problem #1

- **Find range**

$X = v_0 t$

$= (3.23)(.91)$

$X = 2.94\ m$




$V_{o(x)} = 3.23$ m/s

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | 3.23 | 4.45 |
| v | 3.23 | -4.45 |
| a | 0 | -9.8 |
| t | .91 | .91 |
| $t_{max}$ | — | .455 |
| $\Delta y_{max}$ | — | 1.01 |

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# Angled Launch Problem #2

- **An arrow is shot into the air at 7.5 m/s, at an angle of $36^0$. Find:**
  - > **the time in the air.**
  - > **where the target should be placed, assuming it is at the same height as the arrow was shot.**
  - > **the maximum height the arrow reaches.**

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | | |
| v | | |
| a | 0 | -9.8 |
| t | | |
| $t_{max}$ | — | |
| $\Delta y_{max}$ | — | |

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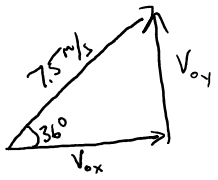
# Angled Launch Problem #2

① 


$V_{ox} = 7.5\cos 36°$
$= 6.07\ m/s$

$V_{oy} = 7.5\sin 36°$
$= 4.41\ m/s$

② FIND TIME

$V_y = V_{oy} + At$

$-4.41 = 4.41 + (-9.8)t$

$-8.82 = -9.8t$

$\frac{-8.82}{-9.8} = t$

$t = .9s$

③ FIND RANGE

$x = V_{ox}t$

$= (6.07)(.9)$

$= 5.46m$

④ FIND $y_{MAX}$

$y_{max} = y_o + v_{oy}t_{max} + \frac{1}{2}a_y t^2_{max}$

$= 0 + 4.41(.45) + \frac{1}{2}(-9.8)(.45)^2$

$y_{max} = .99m$

| | X | Y |
|---|---|---|
| x or y | | 0 |
| $x_o$ or $y_o$ | 0 | 0 |
| $v_0$ | 6.07 | 4.41 |
| v | 6.07 | -4.41 |
| a | 0 | -9.8 |
| t | .9 | .9 |
| $t_{max}$ | — | .45 |
| $\Delta y_{max}$ | — | .99m |

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