

Projectile Motion

Projectile Motion

- **Projectiles**
 - > objects given an initial velocity that then move under the force of gravity
- **Trajectory**
 - > the path followed by a projectile

Independence of Dimensions



Why do both balls hit at the same time?

Independence of Dimensions

- Since the horizontal and vertical motion of an object are independent of each other, the motion equations can be used to determine the exact position of a projectile.
- However, we must first distinguish between the x and y components of any vectors.

Independence of Dimensions

- With no acceleration in the horizontal direction, we can find the horizontal position by using the equation:
 - > $x = v_{0(x)}t + \frac{1}{2} a_x t^2$
- The velocity in the horizontal direction will not change, therefore:
 - > $v_x = v_{0(x)} + a_x t$

Independence of Dimensions

- Since there is acceleration (gravity) in the vertical direction the position can be found using the equation:
 - > $y = v_{0(y)}t + \frac{1}{2} a_y t^2$
- The acceleration causes a change in velocity in the vertical direction. We can find the final velocity using the equation:
 - > $v_{(y)} = v_{0(y)} + a_y t$

Projectile Problem

- A stone is thrown horizontally at a speed of 5 m/s from the top of a cliff 78.4 m high.
 - > How long is the stone in the air?
 - > How far from the cliff does the stone land?
 - > What is the horizontal and vertical components of the velocity just before the stone hits the ground?

Projectile Problem

- Find the time

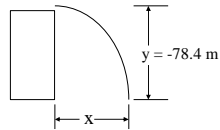
> $y = v_{0(y)}t + \frac{1}{2} a_y t^2$

> $y = \frac{1}{2} a_y t^2$

> $t^2 = 2y / a_y$

> $t^2 = 2(-78.4 \text{ m}) / (-9.8 \text{ m/s}^2) = 16 \text{ s}^2$

> $t = 4 \text{ s}$



Projectile Problem

- Find the horizontal distance

> $x = v_x t$

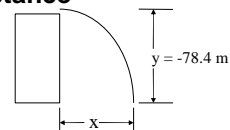
> $x = (5.0 \text{ m/s})(4.00 \text{ s})$

> $x = 20 \text{ m}$

- Find the components

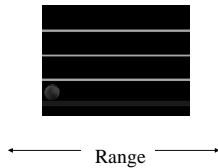
> $v_x = 5.0 \text{ m/s}$

> $v_{(y)} = a_y t = (-9.8 \text{ m/s}^2)(4.00 \text{ s}) = -39.2 \text{ m/s}$



Projectiles Launched at an Angle

- When projectiles are launched at an angle, they are given an initial horizontal and vertical velocity.
- The horizontal distance the projectile travels is called the range.



Angled Launch Problem

- A ball is thrown with a initial velocity of 5.5 m/s at an angle of 54° . Find:
 - > the time in the air.
 - > how high the ball went.
 - > what the range was.

Angled Launch Problem

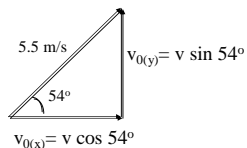
- Find components

$$\Rightarrow v_{0(x)} = 5.5 \cos 54^\circ$$

$$\Rightarrow v_{0(x)} = 3.23 \text{ m/s}$$

$$\Rightarrow v_{0(y)} = 5.5 \sin 54^\circ$$

$$\Rightarrow v_{0(y)} = 4.45 \text{ m/s}$$

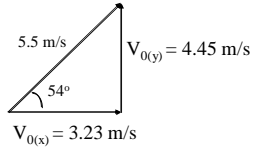


Angled Launch Problem

- Find time at $y = 0$

$$> 0 = v_{0(y)}t + \frac{1}{2} a_y t^2$$

$$> t^2 = -2v_{0(y)} / a_y$$



$$> t^2 = -2(4.45\text{m/s}) / (-9.8 \text{ m/s}^2)$$

$$> t = .91\text{s}$$

Or use the quadratic formula

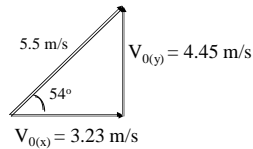
Angled Launch Problem

- Find Max height
- Maximum height occurs at $t / 2$.

$$> y = v_{0(y)} t + \frac{1}{2} a t^2$$

$$> y = (4.45)(.45) + \frac{1}{2} (-9.8)(.45)^2$$

$$> y = 1.01 \text{ m}$$



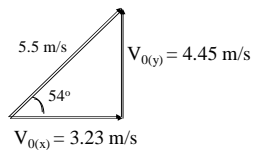
Angled Launch Problem

- Find range

$$> x = v_x t$$

$$> x = (3.23\text{m/s})(.91\text{s})$$

$$> x = 2.9\text{m}$$



Projectiles Launched at an Angle

- It can be proven using trigonometric identities that the range of the projectile can be found using:

$$R = \frac{v_0^2}{g} \sin 2\theta$$

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