Circular Motion

Uniform Circular Motion

- Definition
  - moving in a circle at a constant speed
- Rotating
  - Moving around an axis located within the object itself (ie. spinning top)
- Revolving
  - Moving around an axis located outside the object (ie. Earth around the sun)

Uniform Circular Motion

- Period (T)
  - the amount of time it takes for an object to make one revolution around the circle
- Frequency (*f*)
  - The amount of revolutions or cycle each second
  - Notice the relation between Period and frequency

$$f = \frac{1}{T}$$

Circular (Tangential) Speed

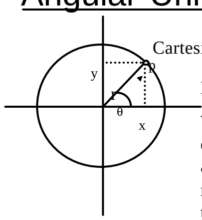
- Velocity of the object moving at a constant rate around a circular path
  - Start with the equation for velocity

$$v = \frac{d}{t}$$

  - Then substitute the values for a circle

$$v = \frac{d}{t} = \frac{2pr}{T}$$

Angular Units



Cartesian Coordinates (x,y)

Polar Coordinates (r,θ)

Where θ is the angular displacement (in radians) and r is the radius (in meters), or distance from the origin

Note: Radians is a dimensionless unit.

- For rotating and revolving situations, it is easier to account for the change in the angle and radius rather than the x and y coordinates

Rotary Motion

- Motion of an object around an internal axis
- Angular Velocity
  - the rate at which the angular displacement changes

$$w = \frac{\Delta q}{\Delta t}$$

  - 1 revolution = 360^0 = 2π radians
    or

$$1 \text{ radian} = \frac{360°}{2p} = 57.3°$$

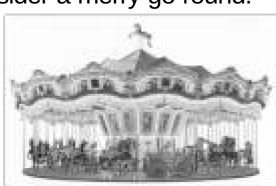
Rotary Motion

- Angular Velocity
  - If we look at an object making complete rotations or revolutions, the angular velocity of the object can be found using:

$$w = \frac{2p}{T} = 2pf$$

Tangential Velocity and Angular Velocity

- Consider a merry go round:



- Where do you have the largest angular velocity?
  - Same at all locations
- Where do you have the largest tangential velocity?
  - The outer edge of the merry go round

Tangential Velocity and Angular Velocity

- Consider a merry go round:

- Based on your previous answers, tangential velocity is related to distance you are from the center of rotation.

$$v = rw$$

Circular Acceleration

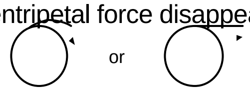
- Centripetal => “center seeking”
- Centripetal Acceleration
  - $a_c = \frac{v^2}{r} = \frac{(rw)^2}{r} = rw^2$
  - $a_c = \frac{v^2}{r} = \frac{\left(\frac{2pr}{T}\right)^2}{r} = \frac{4p^2 r}{T^2}$

Circular Forces

- Centripetal Force
  - force applied to the object to keep it moving in a circle.

$$F_c = ma_c = m\frac{v^2}{r}$$

- What direction will the object move once the centripetal force disappears?



Circular Forces:Application

- A car moves around a curve that has a radius of 45.0 m. If the concrete pavement is dry, what is the maximum speed that the car can move around the curve without skidding?

- What is keeps the car from skidding off the track?
  - Friction

Circular Forces:Application

- So the force of friction must apply the centripetal force or:

$$F_f = F_C \Rightarrow mF_N = \frac{mv^2}{r} \Rightarrow mmg = \frac{mv^2}{r}$$

- Solving for velocity, we get:

$$v^2 = m gr \Rightarrow v = \sqrt{m gr} \Rightarrow v = \sqrt{.8(9.8\text{m/s}^2)45\text{m}}$$

$$v = 18.8 \text{ m/s}$$

Vertical Circles

- What is the minimum velocity required to make it around the top of the circle?
  - At the top of the circle, the centripetal force must be equal to the force due to gravity.

$$m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

Rotary Motion:Acceleration

- Angular Acceleration
  - rate of change of angular velocity

$$a = \frac{\Delta w}{\Delta t}$$

- Angular and Linear Acceleration
  - Like linear velocity, linear acceleration also varies with the distance from the center of motion, therefore:

$$a_t = ra$$

Linear vs. Rotary

- All rotary equations follow the same format as their linear counterparts

Linear	Rotational
$v = v_o + at$	$w = w_o + a t$
$x = v_o t + \frac{1}{2} a t^2$	$q = w_o t + \frac{1}{2} a t^2$
$v^2 = v_o^2 + 2ax$	$w^2 = w_o^2 + 2a q$

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